can be obtained, all weights from 1 to $\frac{3^n-1}{2}$ can be obtained by using the weights 1, 3, 9, 27, ...3^{n-1}.

Also solved by *J. SCHEFFER*, and *J. E. SANDERS*.
No solution of problem 168 has yet been received.

GEOMETRY.

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178. Proposed by JOHN M. ARNOLD, Crompton, R. I.

A cylinder thirty feet long and two feet in diameter is to be placed in a machinery car, the inside dimensions of which are eight feet wide and eight feet high. Find length of shortest car that will contain it.

Solution by the PROPOSER.

Consider the car standing on one end, which we shall call the base. Fig. 2.

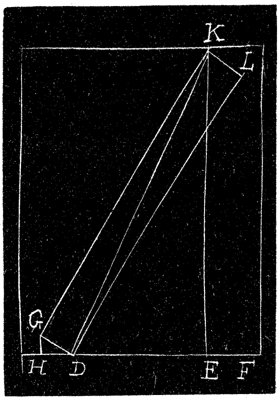

Fig. 1.

If a projection of the cylinder be made on the base, each end will be projected into an ellipse with its minor axis on the diagonal MN of the base.

Fig. 1 is a vertical plane taken on the line MN. [In Fig. 2, the points H, D, E, and F correspond to the same points in Fig. 1.]

As the sides of the square in Fig. 2 are tangents to the equal ellipses, we have by Analytical Geometry $MO=\sqrt{(A^2+B^2)}$, where A and B are the semi-major and semi-minor axes,

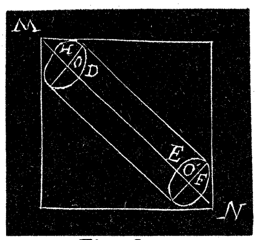

Fig. 2.

$$2\overline{MO}+\overline{OO'}=\overline{MN}=8\sqrt{2}....(1).$$

In the similar triangles GDH and DLF, $GD : HD=DL : LF$, or $2A : 2B=30 : \frac{30B}{A}$. Then $DF=OO'=\sqrt{900-\frac{900B^2}{A^2}}$.

Substituting values of OO' and MO in (1),

$$2\sqrt{(A^2+B^2)}+\sqrt{900-\frac{900B^2}{A^2}}=8\sqrt{2}.$$

A=radius of cylinder=1. Substituting and reducing

$$12769B^4-21728B^2=-9184,$$

from which $B^2=.918904$ and $B=.9586$. Substituting the value of B^2 in the expression for DF, $DF=OO'=8.5432$. Then $DE=OO'-2B=6.626$, and $EK=\sqrt{[(30)^2+(2)^2-(6.626)^2]}=29.3274$ feet, or 29 feet 3.9288 inches, the length of car required.

Also solved by the late *P. H. PHILBRICK*, who obtained as a result, 29.168 feet.

CALCULUS.

155. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College Defiance, Ohio.

Solve the differential equations:

$$(A).\quad \frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}=\sin 2x+\sin x-x. \qquad (B).\quad \frac{d^2y}{dx^2}+2\frac{dy}{dx}=\sin 2x+\sin x-x.$$

Solution by CHRISTIAN HORNUNG, A. M., Heidelberg University, Tiffin, O., and LON C. WALKER, A. M., Leland Stanford University.

Using the symbolic method (A) becomes $(D^4+2D^2)y=\sin 2x+\sin x-x$. The complementary function is $c_1+c_2x+c_3\cos\sqrt{2}x+c_4\sin\sqrt{2}x$, and the particular integral

$$=\frac{1}{D^2(D^2+2)}(\sin 2x+\sin x-x)=\frac{1}{D^2}\cdot\frac{1}{D^2+2}\sin 2x+\frac{1}{D^2}\cdot\frac{1}{D^2+2}\sin x-$$

$$\frac{1}{D^2+2}\cdot\frac{1}{D^2}x=\tfrac{1}{8}\sin 2x-\sin x-(2+D^2)^{-1}\cdot\frac{x^3}{6}=\tfrac{1}{8}\sin 2x-\sin x-\frac{x^3}{12}+\tfrac{1}{4}x.$$

$\therefore\ y=c_1+c_2x+c_3\cos\sqrt{2}x+c_4\sin\sqrt{2}x+\frac{1}{8}\sin 2x-\sin x-\frac{1}{12}x^3$ ($\frac{1}{4}x$ being included in c_2x); and (B) becomes $(D^2+2D)y=\sin 2x+\sin x-x$.

$\therefore$ The complementary function is $c_1+c_2e^{-2x}$, and the particular integral

$$=\frac{1}{D^2+2D}(\sin 2x+\sin x-x)=\frac{1}{D^2+2D}\sin 2x+\frac{1}{D^2+2D}\sin x-\frac{1}{D+2}\cdot\frac{1}{D}x$$

$$=\frac{1}{2D-4}\sin 2x+\frac{1}{2D-1}\sin x-(2+D)^{-1}\cdot\tfrac{1}{2}x^2$$

$$=\tfrac{1}{2}\cdot\frac{D+2}{D^2-4}\sin 2x+\frac{2D+1}{4D^2-1}\sin x-(\tfrac{1}{2}-\tfrac{1}{4}D+\tfrac{1}{8}D^2)\tfrac{1}{2}x^2$$

$$=-\tfrac{1}{16}(D+2)\sin 2x-\tfrac{1}{5}(2D+1)\sin x-\tfrac{1}{4}x^2+\tfrac{1}{4}x-\tfrac{1}{8}$$

$$=-\tfrac{1}{8}\cos 2x-\tfrac{1}{8}\sin 2x-\tfrac{2}{5}\cos x-\tfrac{1}{5}\sin x-\tfrac{1}{4}x^2+\tfrac{1}{4}x-\tfrac{1}{8}.$$

$\therefore\ y=c_1+c_2e^{-2x}-\frac{1}{8}(\cos 2x+\sin 2x)-\frac{2}{5}\cos x-\frac{1}{5}\sin x-\frac{1}{4}x^2+\frac{1}{4}x$, ($-\frac{1}{8}$ being included in the term c_1).

Also solved by *J. SCHEFFER*, *W. W. LANDIS*, *G. W. GREENWOOD*, and *WILLIAM HOOVER*.